

GOSSIP-BASED INFORMATION SPREADING IN MOBILE NETWORKS

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Abstract

Mobile networks receive increasing research interest recently due to their increasingly wide applications in various areas; mobile ad hoc networks (MANET) and Vehicular ad hoc networks (VANET) are two prominent examples. Mobility introduces challenges as well as opportunities: it is known to improve the network throughput as shown in [1]. In this paper, we analyze the effect of mobility on the information spreading based on gossip algorithms. Our contributions are twofold. Firstly, we propose a new performance metric, mobile conductance, which allows us to separate the details of mobility models from the study of mobile spreading time. Secondly, we explore the mobile conductances of several popular mobility models, and offer insights on the corresponding results. Large scale network simulation is conducted to verify our analysis.

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Index Terms

Conductance, Gossip, Information Spreading, Mobile Networks, Mobility Models.

I. INTRODUCTION

A. Motivation

In many real world networks, an interesting application is to broadcast the information from some source node to the whole network. For wireless ad hoc and sensor networks, a node triggered by the event of interest may want to inform the whole network about the situation as quickly as possible. For social networks, rumors and stories are forwarded by people via different communication media. In these and many other applications, how fast a message can be spread to the whole network is of particular interest as opposed to the general network throughput.

Mobility is becoming one key feature in many existing and emerging networks. However, its effect on the information spreading is far less understood. In mobile networks, will the information spreading speed up or slow down? How may the different mobility patterns affect the information spreading? These problems are of major importance and deserves further study.

B. Related Works

Information spreading in static networks has already been well studied in literature [2], [3]. Gossip algorithm, dated back to [4], is a simple but effective fully distributed information spreading strategy, in which every node randomly selects one of its neighbors for message exchange during the information spreading process. The spreading time of gossip algorithms is near-optimal for complete graphs and random geometric graphs [2]. It is also found that the spreading time is closely related to the geometry of the network, defined specifically as “conductance” [5], [6], which represents the bottleneck for information exchange within a network.

Mobile networks has drawn increasing research interest in recent years. Traditionally, mobility is viewed as a negative feature as it adds additional uncertainty to wireless networks, and incurs more challenges in channel estimation. Recently, mobility has been revisited for its potential to improve network performance. In the seminal work [1], mobility is shown to significantly increase the sum-throughput of the network. Subsequently, the throughput-delay tradeoff is further investigated in the context of mobile ad-hoc networks [7]–[10]. Various mobility models have been investigated, including fully random mobility [1], velocity constrained mobility [11], virtual mobility [12], one-dimensional mobility [13] and area constrained mobility [14].

Although there have been extensive study on both information spreading and mobility of networks, separately, only a few works have jointly considered them, i.e. information spreading in mobile networks. A stationary Markovian evolving graph model is introduced in [15], but their focus is on average point to point (P2P) delay. The mobile spreading time is studied for a velocity-limited mobility model in [11], and for a random mobility model with a few virtually mobile agents in [12]. Both of them consider specific mobility models. In [16], mobility is considered in gossip algorithms. However, the focus is on the energy efficiency, rather than the spreading time.

C. Summary of Contributions

In this work, we intend to develop a more general framework for the study of the information spreading in mobile networks. The main contributions of this paper are summarized below.

- 1) Based on a “move-and-gossip” information spreading model, we propose a new metric, mobile conductance, which represents the capability of a mobile network to conduct information flows. Mobile conductance is dependent not only on the network geometry structure, but also on the mobility patterns. Facilitated by the definition of mobile conductance, the mobile spreading time for a generic mobility model is derived.
- 2) We evaluate the mobile conductances for various mobility models, including fully random mobility, partially random mobility, velocity constrained mobility, one-dimensional and two-dimensional area constrained mobility, and offer insights on the results. The results are summarized in Table. I. In particular, the study on the fully random mobility model reveals that the potential improvement in information spreading time due to mobility is dramatic: from $\Theta(\sqrt{n})$ to $\Theta(\log n)$. We have also carried out large scale simulations to verify our analysis on mobile conductance and information spreading time.

II. PROBLEM FORMULATION

A. System Model

We consider an n -node mobile network $G(V(t), E(t))$. $E(t) \subset V(t) \times V(t)$ denotes the set of time-varying links between the node pairs for communications, and the change is assumed to happen over discrete intervals. Denote all realizations of a geometric random graph [17] with n nodes and transmission range r as set $\mathcal{G}(n, r)$. It is assumed that the mobile network under

TABLE I
CONDUCTANCES OF DIFFERENT MOBILITY MODELS

Static Conductance	$\Phi_s = \Theta\left(\sqrt{\frac{\log n}{n}}\right).$
Mobility Model	Mobile Conductance Φ_m
Fully Random	$\Phi_m = \frac{1}{2} = \Theta(1).$
Partially Random	$\Phi_m = \left(\frac{n-k}{n}\right)^2 \Phi_s + \frac{k(2n-k)}{2n^2}.$
Velocity Constrained	$\Phi_m = \begin{cases} \Theta(r), & v_{\max} \ll r, \\ \Theta(r) \text{ or } \Theta(v_{\max}), & v_{\max} = \Theta(r), \\ \Theta(v_{\max}), & v_{\max} \gg r. \end{cases}$
Area Constrained: One-Dim	$\Phi_m = \frac{n_V^2 + n_H^2}{n^2} \Phi_s + \frac{n_V n_H}{n^2}.$
Area Constrained: Two-Dim	$\Phi_m = \begin{cases} \Theta(r), & r_c \ll r, \\ \Theta(r) \text{ or } \Theta(r_c), & r_c = \Theta(r), \\ \Theta(r_c), & r_c \gg r. \end{cases}$

consideration $G(V(t), E(t)) \in \mathcal{G}(n, r), \forall t \in \mathbb{N}$, which means all nodes are uniformly distributed at each time instant. In static networks, the transmission range r of each node is usually chosen as $\Theta\left(\sqrt{\frac{\log n}{n}}\right)^1$ to ensure connectivity [18]. In mobile networks, such a requirement may be relaxed; but in this work we consider the same r for the mobile network for ease of comparison.

B. Spreading Model: Move-and-Gossip

In this study, we adopt a move-and-gossip model to facilitate our analysis. Specifically, information spreading in a mobile network is decomposed into two steps in each time slot. First, all nodes *move* according to some mobility model (further discussed below), and then each node *gossips* with one of its new neighbors.

Denote $S(t) \subset V(t)$ as the set of nodes that have the message m_s , $X_i(t)$ the position of node i , and $\mathcal{N}_i(t)$ its neighboring set, at the *beginning* of time slot t . A node j belongs to $\mathcal{N}_i(t)$ if $|X_i(t) - X_j(t)| < r$. Initially only the source node s has the message, say m_s , i.e. $S(0) = \{s\}$. The move-and-gossip strategy is illustrated in Fig. 1, where we adopt the following conventions for our notations throughout the paper: $X_i(t)$ changes at the middle of each time slot (after the move step), while $S(t)$ is not updated till the end (after the gossip step). We use $P_{ij}(t+1)$ to

¹Since $r \rightarrow 0$ as $n \rightarrow \infty$, we ignore the edge effect in this study.

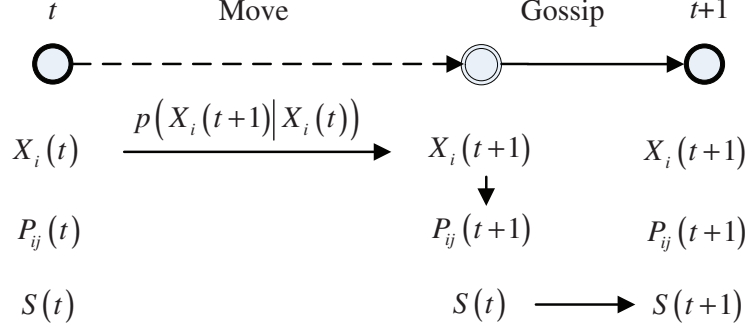


Fig. 1. Move-and-Gossip Spreading Strategy

denote the the probability that node i contacts one of its *new* neighbors $j \in \mathcal{N}_i(t+1)$ in the gossip step of slot t . Without loss of generality, $P_{ij}(t+1)$ is set as $1/|\mathcal{N}_i(t+1)|$ for $j \in \mathcal{N}_i(t+1)$, and 0 otherwise. During the gossip step, it is assumed that each node only contacts one of its neighbors, and the the message is successfully delivered for each meaningful contact.

The positions of all nodes in the network are modeled as a (second-order strict-sense) stationary and ergodic random process $\{\mathcal{P}_{\mathbf{X}}(\cdot)\}$, and a random move is characterized by the state transition probability distribution $p(\mathbf{X}(t+1)|\mathbf{X}(t))$.

Our performance metric is the mobile spreading time, defined as:

$$T_{mobile}(\varepsilon) = \sup_{s \in V(0)} \inf \{t : \Pr(|S(t)| \neq n | S(0) = \{s\}) \leq \varepsilon\}. \quad (1)$$

III. MOBILE CONDUCTANCE

A. Preliminaries on Static Networks

We first recall some relevant results in static networks. According to [2], the static spreading time is

$$T_{static}(\varepsilon) = O\left(\frac{\log n + \log \varepsilon^{-1}}{\Phi_s}\right), \quad (2)$$

where Φ_s is the static conductance defined as

$$\Phi_s \triangleq \min_{S \subset V, |S| \leq n/2} \left(\frac{\sum_{i \in S, j \in \bar{S}} P_{ij}}{|S|} \right). \quad (3)$$

Discussions: For the network model considered, $P_{ij} = \Theta\left(\frac{1}{n\pi r^2}\right) = \Theta\left(\frac{1}{\log n}\right)$ when $j \in \mathcal{N}_i$. Since the focus is on the scaling law study, such P_{ij} may be treated as a constant $P(r)$. We name

the (i, j) pairs satisfying $i \in S, j \in \bar{S}$ and $j \in \mathcal{N}_i$ as “contact pairs” and their located region as the “contact region”. The number of contact pairs is defined as $N_S \triangleq \sum_{i \in S, j \in \bar{S}} I_{ij}$, where I_{ij} is the indicator function for the event that i and j are neighbors. Thus, the static conductance may be rewritten as

$$\Phi_s = \min_{S \subset V, |S| \leq n/2} \left(\frac{P(r) N_S}{|S|} \right). \quad (4)$$

It has been shown that the conductance for a static random geometric graph scales as $\Theta(r)$ [5], i.e. $\Phi_s = \Theta\left(\sqrt{\frac{\log n}{n}}\right)$, and the static spreading time scales as

$$T_{static} = O\left(\frac{\log n}{\sqrt{\log n/n}}\right) \approx O(\sqrt{n}).$$

It is worth mentioning that the above result is actually tight in the order sense. The network radius is on the order of $\Theta(1)$, and the distance of one-hop transmission is $\Theta\left(\sqrt{\frac{\log n}{n}}\right)$. Thus, the minimal number of hops is on the order of $\Theta\left(\sqrt{\frac{n}{\log n}}\right) \approx \Theta(\sqrt{n})$. This indicates that the spreading time in the static network scales as $\Theta(\sqrt{n})$.

B. Mobile Conductance and Mobile Spreading Time

Conductance essentially determines the network bottleneck in information spreading. Intuitively, node movement introduces dynamics into the network structure, thus can facilitate the information flows. In this work we define a new metric, *mobile conductance*, to measure and quantify such improvement.

Given an arbitrary node set $S'(t)$, we denote the number of contact pairs between $S'(t)$ and $\overline{S'(t)}$ after the move as $N_{S'}(t+1) = \sum_{i \in S'(t), j \in \overline{S'(t)}} I_{ij}(t+1)$, where $I_{ij}(t+1)$ is the indicator function for the event that node i and j are neighbors after the move and before the gossip step in slot t .

Definition: The mobile conductance of a mobile network under the mobility model $\mathcal{P}_{\mathbf{X}}$ is defined as:

$$\begin{aligned} & \Phi_m(\mathcal{P}_{\mathbf{X}}) \\ & \triangleq \min_{\substack{S'(t) \subset V(t) \\ |S'(t)| < n/2}} \left\{ E_{p(\mathbf{X}(t+1)|\mathbf{X}(t))} \left(\frac{\sum_{\substack{i \in S'(t) \\ j \in \overline{S'(t)}} P_{ij}(t+1)}{|S'(t)|} \right) \right\} \end{aligned}$$

$$= \min_{\substack{S'(t) \subset V(t) \\ |S'(t)| \leq n/2}} \left\{ \frac{P(r)}{|S'(t)|} E_{p(\mathbf{X}(t+1)|\mathbf{X}(t))} [N_{S'}(t+1)] \right\}. \quad (5)$$

Remarks: 1) Different from the static case, in mobile conductance we consider the expected number of contact pairs on the move, with respect to the state transition probability distribution $p(\mathbf{X}(t+1)|\mathbf{X}(t))$, determined by the mobility model and assumed stationary in this work. Due to stationarity, the evaluation of mobile conductance may be restricted to one slot (which contains both the move and the gossip step). In particular, we consider an arbitrary set $S'(t)$ *before the move*, and evaluate the expected number of contact pairs *after the random move* (normalized to the size of $S'(t)$). The mobile conductance is defined with respect to the $S'(t)$ that leads to the minimum of such evaluation.

2) Due to the assumption that nodes are uniformly distributed in each time instant, P_{ij} may be considered as a constant $P(r)$ if $j \in \mathcal{N}_i$, as in the static case.

Based on the above definition, we have the mobile counterpart of (2) as shown below.

Theorem 1: For a mobile network with mobile conductance Φ_m , the mobile spreading time scales as

$$T_{\text{mobile}}(\varepsilon) = O\left(\frac{\log n + \log \varepsilon^{-1}}{\Phi_m}\right). \quad (6)$$

Proof: We follow the two-phase strategy in [2] to estimate the spreading time. In *phase 1*, for each node $j \in \overline{S(t)}$, defines a random variable $I_j(t)$. If at least one node with the message moves to j 's neighboring area in slot t and “pushes” the message to j in the gossip step, $S(t+1)$ will increase by one. We let $I_j(t+1) = 1$ in this case, and 0 otherwise.

Under a given mobility model $\mathcal{P}_{\mathbf{X}}$, we have

$$\begin{aligned} E[I_j(t+1)|S(t)] &= E_{p(\mathbf{X}(t+1)|\mathbf{X}(t))} \left[1 - \prod_{i \in S(t)} (1 - P_{ij}(t+1)) \right] \\ &\geq E_{p(\mathbf{X}(t+1)|\mathbf{X}(t))} \left[1 - \prod_{i \in S(t)} \exp(-P_{ij}(t+1)) \right] \\ &\geq \frac{P(r)}{2} E_{p(\mathbf{X}(t+1)|\mathbf{X}(t))} \left[\sum_{i \in S(t)} I_{ij}(t+1) \right], \end{aligned}$$

where the first and the second inequalities are due to the facts of $1 - x < \exp(-x)$ for $x \geq 0$ and $1 - \exp(-x) \geq \frac{x}{2}$ for $0 \leq x \leq 1$, respectively.

The expected value of the total increment of $|S|$ from time t to $t + 1$ is then given by

$$\begin{aligned}
& E[|S(t+1)| - |S(t)| | S(t)] \\
&= \sum_{j \in \overline{S(t)}} E[I_j(t+1) | S(t)] \\
&\geq \frac{P(r)}{2} E_{p(\mathbf{X}(t+1)|\mathbf{X}(t))} \left[\sum_{i \in |S(t)|, j \in \overline{S(t)}} I_{ij}(t+1) \right] \\
&= \frac{|S(t)|}{2} E_{p(\mathbf{X}(t+1)|\mathbf{X}(t))} \left[\frac{P(r)}{|S(t)|} N_S(t+1) \right] \\
&\geq \frac{|S(t)|}{2} \min_{\substack{S'(t) \subset V(t) \\ |S'(t)| \leq n/2}} \left\{ \frac{P(r)}{|S'(t)|} E_{p(\mathbf{X}(t+1)|\mathbf{X}(t))} [N_{S'}(t+1)] \right\} \\
&= \frac{|S(t)|}{2} \Phi_m(\mathcal{P}_{\mathbf{X}}). \tag{7}
\end{aligned}$$

The form of (7) is consistent with the set increment in static networks [2], [3]. Moreover, *phase 2* and *phase 1* are symmetric. Therefore, we can follow the same procedures of [2] in the rest part of the proof, and the mobile spreading time with our move-and-gossip model is obtained by replacing the static conductance Φ_s in (2) with the mobile conductance $\Phi_m(\mathcal{P}_{\mathbf{X}})$. ■

C. Mobility Enhances Conductivity

Our following result shows that in general mobile conductance is no smaller than its static counterpart, and thus mobility generally speeds up the information spreading.

Proposition 1: For a mobile network represented by a time-varying graph satisfying $G(V(t), E(t)) \in \mathcal{G}(n, r), \forall t \in \mathbb{N}$, its mobile conductance Φ_m is no smaller than the static conductance Φ_s of any static graph $G(V, E) \in \mathcal{G}(n, r)$ for sufficiently large n .

Proof: For a static graph $G(V, E) \in \mathcal{G}(n, r)$, the segmentation of V into two subsets S_{bn} and $\overline{S_{bn}}$ is called the *bottleneck segmentation* if the following is satisfied

$$S_{bn} = \arg \min_{S \subset V, |S| \leq n/2} \left(\frac{P(r) N_S}{|S|} \right). \tag{8}$$

It is proved in [5] that for any $G(V, E) \in \mathcal{G}(n, r)$ with sufficiently large n , S_{bn} is comprised of the nodes on the left half of the unit square, after the unit square is bisected by a vertical *straight*

line, and the resulted conductances are approximately the same for all $G(V, E) \in \mathcal{G}(n, r)$:

$$\Phi_s(\mathcal{G}(n, r)) \approx \frac{4r(1-r)}{3\pi}. \quad (9)$$

According to the definition of the mobile conductance, given an arbitrary node set $S'(t)$ before the move, we need to compute the expected number of contact pairs between it and its complementary set after the move.

Since $G(V(t), E(t)) \in \mathcal{G}(n, r), \forall t \in \mathbb{N}$, the minimum of (9) also holds for $G(V(t), E(t))$:

$$\frac{P(r)}{|S'(t)|} N_{S'}(t+1) \geq \frac{P(r)}{|S_{bn}|} N_{S_{bn}} = \Phi_s(\mathcal{G}(n, r)), \quad \forall \{S'(t) \subset V(t), |S'(t)| < n/2\}. \quad (10)$$

After the expectation in (5), a no smaller mobile conductance is ensured.

$$\begin{aligned} \Phi_m(\mathcal{P}_{\mathbf{X}}) &= \min_{\substack{S'(t) \subset V(t) \\ |S'(t)| \leq n/2}} \left\{ \frac{P(r)}{|S'(t)|} \mathbb{E}_{p(\mathbf{X}(t+1)|\mathbf{X}(t))} [N_{S'}(t+1)] \right\} \\ &\geq \Phi_s. \end{aligned} \quad (11)$$

■

Remarks: Due to the random nature of node movement, the new structure of $S'(t)$ will not always fall into the worst category (the above mentioned bottleneck segmentation and the like)², which means the equalities in (10) and (11) are not achieved in most cases. Except for some degenerate mobility models, in which the structure of $S'(t)$ always forms the bottleneck segmentation after the move, the mobile conductance will generally be larger.

The benefit of a mobile network is that its network structure is constantly changing, thus avoiding being stuck at the bottleneck of a corresponding static network. In the following section, we will quantify the performance gain for several mobility models in literature.

IV. EVALUATION OF SEVERAL MOBILITY MODELS

The general definition of mobile conductance allows us to separate the details of mobility models from the study of mobile spreading time. In this section, we will evaluate the mobile conductances of several popular mobility models, including fully random mobility [1], partially random mobility, velocity constrained mobility [11], and one/two dimensional area constrained mobility [13] [14]. The main efforts lie in finding the bottleneck segmentation, and then determining the expected number of contact pairs.

²As we will see in the following section, in many cases the improvement due to mobility is substantial.

A. Fully Random Mobility

To evaluate the maximal improvement introduced by mobility, we first investigate an extreme case called the fully random mobility model [1]. In this model, node positions are uniformly distributed on the unit square and i.i.d over time.

Theorem 2: In fully random mobile networks, the mobile conductance scales as $\Theta(1)$, and the corresponding mobile spreading time scales as $O(\log n)$.

Proof: According to the definition of mobile conductance, Φ_m is calculated over all possible node movement. Since the mobility model $\mathcal{P}_{\mathbf{X}}$ is memoryless, for an arbitrary $S'(t)$, the nodes in both $S'(t)$ and $\overline{S'(t)}$ are uniformly distributed after the move.

For each node in $S'(t)$, the size of its neighborhood area is πr^2 , and the density of nodes in $\overline{S'(t)}$ is $\frac{|\overline{S'(t)}|}{|V(t)|}n$. Therefore, the expected number of contact pairs is

$$\begin{aligned} & E_{p(\mathbf{X}(t+1)|\mathbf{X}(t))} [N_{S'}(t+1)] \\ &= |S'(t)| \frac{|\overline{S'(t)}|}{|V(t)|} n \pi r^2 = |S'(t)| |\overline{S'(t)}| \pi r^2. \end{aligned}$$

According to the definition of mobile conductance,

$$\begin{aligned} \Phi_m(\mathcal{P}_{\mathbf{X}}) &= \min_{\substack{S'(t) \subset V(t) \\ |S'(t)| \leq n/2}} \left\{ \frac{P(r)}{|S'(t)|} E_{p(\mathbf{X}(t+1)|\mathbf{X}(t))} [N_{S'}(t+1)] \right\} \\ &= \min_{S'(t) \subset V(t), |S'(t)| \leq \frac{n}{2}} \left(P(r) |\overline{S'(t)}| \pi r^2 \right) \\ &= \frac{1}{2} = \Theta(1). \end{aligned} \tag{12}$$

Substituting the mobile conductance into *Theorem 1*, we can see the spreading time of the fully random mobile network scales as $O(\log n)$. ■

Remarks: In the gossip algorithms, only the nodes with the message can contribute to the increment of $|S(t)|$. Consider the ideal case that each node with the message contacts a node without message in each step, which represents the fastest possible information spreading. We have the following straightforward arguments:

$$\begin{aligned} & |S(t+1)| - |S(t)| \leq |S(t)| \\ \Rightarrow & |S(t+1)| \leq 2|S(t)| \\ \Rightarrow & |S(t)| \leq 2^t = O(e^t). \end{aligned}$$

When $|S(T)|$ reaches $(1 - \epsilon)n$, the message has largely been spread to the whole network. Therefore, $T_{mobile}(\epsilon) = \Omega(\log n)$ for arbitrary constant ϵ , and the optimal performance in information spreading is achieved in the fully random model. The potential improvement on information spreading time due to mobility is dramatic: from $\Theta(\sqrt{n})$ to $\Theta(\log n)$.

B. Partially Random Mobility

In many existing mobile networks, only a portion of the nodes are mobile while many other nodes stay static. Social networks are such examples. For the most time, the majority of people will stay in the same place, such as their hometowns. These people only exchange information with their neighbors. Only a few people, such as diplomats or business men, travel very often, spreading information to faraway places. In other words, not all real world networks are fully mobile, and this motivate us to analyze the information spreading time under the ‘partially random mobility’ model.

In the partially random mobility model, we assume that only k out of n nodes are mobile, while the rest $n - k$ nodes stay static. The k mobile nodes are randomly and independently chosen. The mobility pattern of the k mobile nodes follows the fully random mobility model.

Theorem 3: For the partially random mobility model, where k out of n nodes are fully mobile, and the rest $n - k$ nodes stay static, the mobile conductance Φ_m scales as $\left(\frac{n-k}{n}\right)^2 \Phi_s + \frac{k(2n-k)}{2n^2}$.

Proof: For each node that already has the message, say i , among all its neighbors, there are on average $(n - k) \pi r^2$ static nodes and $k \pi r^2$ mobile nodes. We denote the set of k mobile (dynamic) nodes at time t as $D(t)$, the set of $n - k$ static nodes at time t as $\overline{D}(t)$ and calculate the number of contacted pairs separately.

$$\begin{aligned}
 & E_{p(\mathbf{X}(t+1)|\mathbf{X}(t))} [N_{S'}(t+1)] \\
 &= E_{p(\mathbf{X}(t+1)|\mathbf{X}(t))} \left[\begin{aligned} & \sum_{i \in S'(t) \cap \overline{D}(t), j \in \overline{S'}(t) \cap \overline{D}(t)} I_{ij}(t+1) \\ & + \sum_{i \in S'(t) \cap D(t), j \in \overline{S'}(t) \cap D(t)} I_{ij}(t+1) \\ & + \sum_{i \in S'(t) \cap D(t), j \in \overline{S'}(t) \cap \overline{D}(t)} I_{ij}(t+1) \\ & + \sum_{i \in S'(t) \cap \overline{D}(t), j \in \overline{S'}(t) \cap D(t)} I_{ij}(t+1) \end{aligned} \right] \quad (13)
 \end{aligned}$$

where the former two terms are the number of contact pairs *within* static nodes and mobile nodes, the latter two terms are the number of contact pairs *between* static and mobile nodes.

The links within the static nodes remain unchanged after the move, therefore $I_{ij}(t+1) = I_{ij}(t)$ for $i \in S'(t) \cap D(t)$, $j \in \overline{S'(t)} \cap D(t)$. Since the k mobile nodes are fully random, the links involving the mobile nodes can be estimated similarly as in the fully random model. Take the third term for example, for each static node without message (total $\frac{n-k}{n} |\overline{S'(t)}|$), there are $n\pi r^2 \frac{|S'(t)|}{n} \frac{k}{n}$ neighboring mobile nodes with message. Putting together (with some reorganization), we have

$$\begin{aligned} & \mathbb{E}_{p(\mathbf{X}(t+1)|\mathbf{X}(t))} [N_{S'}(t+1)] \\ &= \mathbb{E}_{p(\mathbf{X}(t+1)|\mathbf{X}(t))} \left[\sum_{\substack{i \in S'(t) \cap \overline{D(t)}, \\ j \in \overline{S'(t)} \cap \overline{D(t)}}} I_{ij}(t) \right] + \left(\begin{aligned} & \frac{n-k}{n} |\overline{S'(t)}| n\pi r^2 \frac{|S'(t)|}{n} \frac{k}{n} \\ & + \frac{n-k}{n} |S'(t)| n\pi r^2 \frac{|\overline{S'(t)}|}{n} \frac{k}{n} \\ & + \frac{k}{n} |S'(t)| n\pi r^2 \frac{|\overline{S'(t)}|}{n} \frac{k}{n} \end{aligned} \right) \end{aligned} \quad (14)$$

According to the definition of mobile conductance,

$$\begin{aligned} \Phi_m(\mathcal{P}_{\mathbf{X}}) &= \min_{S'(t) \subset V(t), |S'(t)| \leq n/2} \left\{ \frac{P(r)}{|S'(t)|} \sum_{\substack{i \in S'(t) \cap \overline{D(t)}, \\ j \in \overline{S'(t)} \cap \overline{D(t)}}} I_{ij}(t) \right\} \\ &+ \min_{S'(t) \subset V(t), |S'(t)| \leq n/2} \left\{ 2 \frac{n-k}{n} \frac{|\overline{S'(t)}|}{n} \frac{k}{n} + \frac{k}{n} \frac{|\overline{S'(t)}|}{n} \frac{k}{n} \right\} \\ &= \left(\frac{n-k}{n} \right)^2 \Phi_s + \frac{k(2n-k)}{2n^2}. \end{aligned} \quad (15)$$

Note that the two minima are achieved simultaneously when $|S'(t)| = |\overline{S'(t)}| = \frac{n}{2}$. ■

Remarks: Since $\Phi_s = \Theta(\sqrt{\log n/n})$ and $\frac{1}{2} \frac{k}{n} < \frac{k(2n-k)}{2n^2} < \frac{k}{n}$, the number of mobile nodes needs to achieve $\omega(\sqrt{n \log n})$ in order to bring significant benefit over the static one. Partially mobility model is a mixture of the static network and fully mobile network. It can be seen that as k grows, the mobile conductance increases: as $k \rightarrow \Theta(n)$, $\Phi_m \rightarrow \Theta(1)$.

C. Velocity Constrained Mobility

Fully random mobility model implies that the nodes can appear at any positions on the unit square in each time slot, which may not be realistic. Usually, the speed of a mobile node is constrained by both its limited hardware and power. Therefore, we describe a velocity constrained

mobility model, which is similar to the mobility model in [14], and analyze its influence on the mobile conductance and corresponding information spreading speed.

The node speed is defined as its moving distance per slot:

$$v(t) \triangleq |X_i(t+1) - X_i(t)| \leq v_{\max},$$

where v_{\max} is the velocity constraint. Without loss of generality, we assume that after the move, the new positions are uniformly distributed within a circle of radius v_{\max} centered at its original position.

Theorem 4: For the mobility model with velocity constraint v_{\max} , when $v_{\max} = w(r)$, the mobile conductance scales as $\Theta(v_{\max})$, and when $v_{\max} = o(r)$, the mobile conductance scales as $\Theta(r)$. If $v_{\max} = \Theta(r)$, then mobile conductance scales both as $\Theta(v_{\max})$ and $\Theta(r)$.

Proof: Given arbitrary $S'(t)$ satisfying $|S'(t)| = n_0 < n/2$, it is necessary to minimize $E[N_{S'}(t+1)]$ in order to achieve the minimum of mobile conductance defined in (5).

The expected number of contact pairs after the move can be represented as

$$\begin{aligned} E[N_{S'}(t+1)] &= E \left[\sum_{i \in S'(t), j \in V(t)} I_{ij}(t+1) - \sum_{i \in S'(t), j \in S'(t)} I_{ij}(t+1) \right] \\ &= |S'(t)| n \pi r^2 - E \left[\sum_{i \in S'(t), j \in S'(t)} I_{ij}(t+1) \right]. \end{aligned}$$

The first term equals to $n_0 n \pi r^2$ and can be treated as a constant. Therefore minimizing $E[N_{S'}(t+1)]$ is equivalent to maximizing the second term. To be specific, the second term $E[I_{ij}(t+1)] > 0$ only if i and j can possibly move to positions within a distance of r , i.e., $|X_i(t) - X_j(t)| < 2v_{\max} + r$, and the maximum is reached when the number of such node pairs in $S'(t)$ is maximized. Therefore, the nodes in $S'(t)$ should be placed as close as possible, until forming a continuous block. Then there will be a border between $S'(t)$ and $\overline{S'(t)}$, and the nodes at least $2v_{\max} + r$ away from the border cannot have meaningful contact after the move.

According to *Lemma 1* (See Appendix A), the bottleneck segmentation between $S'(t)$ and $\overline{S'(t)}$ is the straight line bisection and the density of nodes before and after move is illustrated in Fig. 2. The darkness of the color represents the density of nodes bearing the message (i.e. in $S'(t)$). We can see that before the move, the nodes in $S'(t)$ and $\overline{S'(t)}$ are strictly separated by a straight line border. After the move, with some nodes in both $S'(t)$ and $\overline{S'(t)}$ crossing the

border to enter the other half, a mixture strip as wide as $2 \times v_{\max}$ emerges in the middle of the graph.

We take the center of the graph as the origin. Denote $\rho_{S'(t)}(l)$ and $\rho_{\overline{S'(t)}}(l)$ as the density of nodes with and without message before moving, and $\rho'_{S'(t)}(l)$ and $\rho'_{\overline{S'(t)}}(l)$ as the density of nodes with and without message after moving, with l the horizontal coordinate. As shown in the upper subfigure of Fig. 2, at time t , the nodes in the circle of radius v_{\max} have equal probabilities to move to the center point at time slot $t + 1$. Therefore, $\rho'_{S'(t)}(l)$ is given by the proportion of dark area in the circle over the total area of the circle (thus is uniform over the vertical line $x = l$). $\rho'_{\overline{S'(t)}}(l)$ can be obtained similarly.

Therefore, for $-v_{\max} < l < v_{\max}$:

$$\begin{aligned}\rho'_{S'}(l) &= \frac{\text{sizeof (dark area in circle)}}{\pi v_{\max}^2} \times n, \\ \rho'_{\overline{S'}}(l) &= \frac{\text{sizeof (white area in circle)}}{\pi v_{\max}^2} \times n.\end{aligned}$$

After some derivation, we have

$$\begin{aligned}\frac{\rho'_{S'}(l)}{n} &= \begin{cases} 1, & l < -v_{\max}, \\ \arccos\left(\frac{l}{v_{\max}}\right) - \frac{l}{v_{\max}} \sin\left(\arccos\frac{l}{v_{\max}}\right), & -v_{\max} < l < v_{\max}, \\ 0, & l > v_{\max}, \end{cases} \\ \frac{\rho'_{\overline{S'}}(l)}{n} &= 1 - \rho'_{S'}(l).\end{aligned}$$

The contact region with the above bottleneck segmentation is the $2 \times (v_{\max} + r)$ wide vertical strip in the center. All nodes outside this region will not contribute to $N_{S'}(t + 1)$.

The number of contact pairs after the move can be calculated according to Fig. 3. The center of the circle with radius r is x away from the middle line. For node i located at the center, the number of nodes that it can push message to is equal to the number of nodes without message in the circle. Since the density of nodes without message at positions l away from the middle line is $\rho'_{\overline{S'}}(l)$, the number of nodes that i can ‘push’ information to is $\int_{x-r}^{x+r} \rho'_{\overline{S'}}(l) 2\sqrt{r^2 - (l - x)^2} dl$. Taking all nodes with message in the contact region into consideration, the expected number of contact pairs after the move is

$$E_{p(\mathbf{X}(t+1)|\mathbf{X}(t))} [N_{S'}(t + 1)]$$

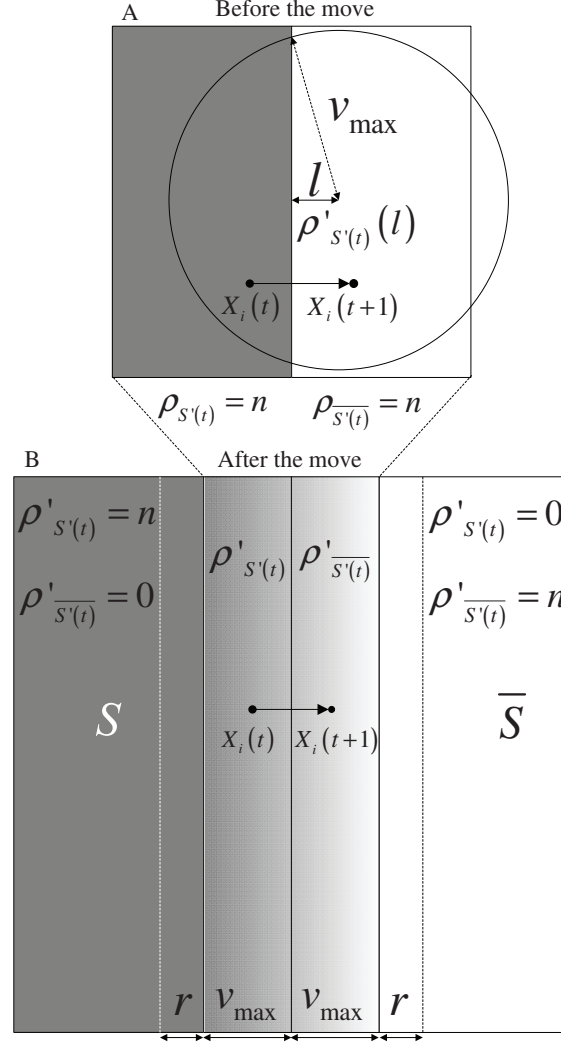


Fig. 2. Velocity Constrained Mobility

$$= \int_{-v_{\max}-r}^{v_{\max}+r} \rho'_{S'}(x) \int_{x-r}^{x+r} \rho'_{\overline{S'}}(l) 2\sqrt{r^2 - (l-x)^2} dl dx. \quad (16)$$

Since $S'(t)$ and $\overline{S'(t)}$ here is the bottleneck segmentation that minimize the conductance, the mobile conductance is $\Phi_m(\mathcal{P}_{\mathbf{X}}) = \frac{2}{n^2 \pi r^2} E_{p(\mathbf{X}(t+1)|\mathbf{X}(t))} [N_{S'}(t+1)]$. According to the calculation in Appendix B, we can obtain the results in *Theorem 4*. ■

Remarks: From (19), we can see that if the node speed is much greater than the transmission radius, then the mobile conductance is dominated by the node speed, i.e. $\Phi_m = \Theta(v_{\max})$. If the node speed is much smaller than the transmission radius, then the mobile conductance is

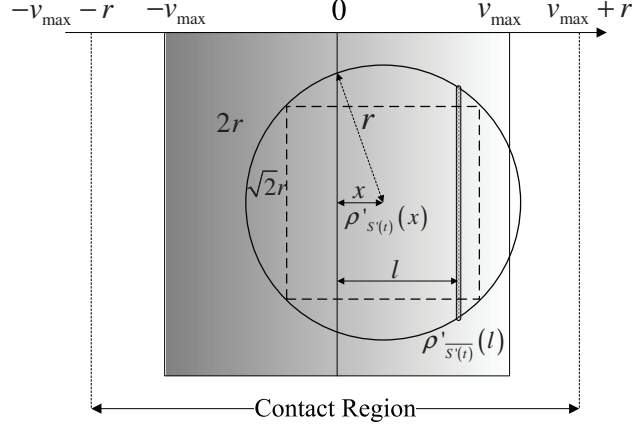


Fig. 3. Calculating the Number of Contact Pairs in Velocity Constrained Mobility

dominated by the transmission radius, i.e. $\Phi_m = \Theta(r)$, which degrades to static networks.

D. Area Constrained Mobility

Many real world applications require that the mobile terminals move along certain paths or move within specified regions. For example, trains and automobiles tend to move along fixed paths, which can be railways or highways. Mobile sensor nodes usually patrol within a certain range from their base positions. While both of them move within constrained areas, the difference is that for the former, the constrained areas are one-dimensional lines, and for the latter, the constrained areas are two-dimensional disks centered at certain points. In this subsection, we define these two kinds of mobility as area constrained mobility. Specifically, we separate them into two subcategories: one-dimensional mobility and two-dimensional mobility. Note that the one-dimensional mobility model had been mentioned in [13] and the two-dimensional mobility model is similar to the general mobility model in [14]. However, our emphasis is on analyzing the effects of mobility on the mobile conductance and the related spreading time, rather than the throughput of the network.

1) *One-dimensional Mobility*: In this model, the mobile nodes move either vertically or horizontally. Among the n nodes, there are n_V vertically moving nodes, named V-nodes, and n_H horizontally moving nodes, named H-nodes. We denote the subset of V-nodes as S_V , and the subset of H-nodes as S_H :

$$|S_V| = n_V, |S_H| = n_H.$$

Both V-nodes and H-nodes are uniformly and randomly distributed on the unit square. It is assumed that the mobility pattern of each node is “fully random” on the corresponding one-dimensional path, i.e., uniformly distributed and i.i.d. over time, and independent with each other.

Theorem 5: For the one-dimensional area constrained random mobility model, where among the n nodes, n_V nodes only move vertically and n_H nodes only move horizontally. The mobile conductance Φ_m scales as $\frac{n_V^2 + n_H^2}{n^2} \Phi_s + \frac{n_V n_H}{n^2}$.

Proof: We consider the V-nodes and H-nodes separately as follows:

$$\Phi_m(\mathcal{P}_{\mathbf{X}}) = \min_{\substack{S'(t) \subset V(t) \\ |S'(t)| < n/2}} \left\{ \frac{P(r)}{|S'(t)|} E_{p(\mathbf{X}(t+1)|\mathbf{X}(t))} \left[\sum_{i \in S_V \cap S'(t), j \in S_V \cap \overline{S'(t)}} I_{ij}(t+1) + \sum_{i \in S_H \cap S'(t), j \in S_H \cap \overline{S'(t)}} I_{ij}(t+1) + \sum_{i \in S_V \cap S'(t), j \in S_H \cap \overline{S'(t)}} I_{ij}(t+1) + \sum_{i \in S_H \cap S'(t), j \in S_V \cap \overline{S'(t)}} I_{ij}(t+1) \right] \right\} \quad (17)$$

The former two terms, $\sum_{i \in S_V \cap S'(t), j \in S_V \cap \overline{S'(t)}} I_{ij}(t+1)$ and $\sum_{i \in S_H \cap S'(t), j \in S_H \cap \overline{S'(t)}} I_{ij}(t+1)$, are the number of contact pairs within V-nodes and H-nodes, respectively. And the latter two terms,

$\sum_{i \in S_V \cap S'(t), j \in S_H \cap \overline{S'(t)}} I_{ij}(t+1)$ and $\sum_{i \in S_H \cap S'(t), j \in S_V \cap \overline{S'(t)}} I_{ij}(t+1)$, are the number of contact pairs between V-nodes and H-nodes. Similarly, we need to find the bottleneck segmentation that minimize the number of contact pairs.

First, we focus on the former two terms. Take the first one for example, i.e. minimizing $N_S^{V-V}(t+1) \triangleq \sum_{i \in S_V \cap S'(t), j \in S_V \cap \overline{S'(t)}} I_{ij}(t+1)$. Because all nodes follow a one-dimensional “fully random” mobility model on their corresponding vertical paths, the number of contact pairs remains unchanged after the move, i.e. $N_S^{V-V}(t+1) = N_S^{V-V}(t)$. Therefore, we equivalently find the bottleneck segmentation before the move, which is the same as in static networks, i.e. letting all V-nodes on the left half belong to $S'(t)$ and those on the right half belong to $\overline{S'(t)}$. However, the density of the V-nodes on both halves is n_V , instead of n , therefore the number of contact pairs in V-nodes is

$$N_S^{V-V}(t+1) = \left(\frac{n_V}{n}\right)^2 N_S,$$

where N_S is the number of contact pairs in a static network with n nodes.

Analogously, $\sum_{i \in S_H \cap S'(t), j \in S_H \cap \overline{S'(t)}} I_{ij}(t+1)$ is minimized by letting all H-nodes on the upper half belong to $S'(t)$ and those on the bottom half belong to $\overline{S'(t)}$, and the number of contact pairs in H-nodes is $\left(\frac{n_H}{n}\right)^2 N_S$.

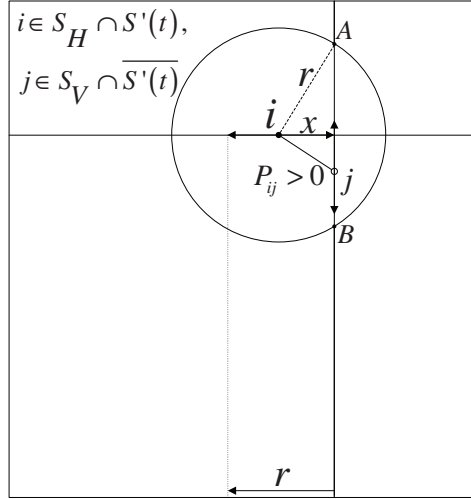


Fig. 4. One-dimensional Area Constrained Mobility

Now we move on to the latter two terms, i.e. minimizing $\sum_{i \in S_V \cap S'(t), j \in S_H \cap \overline{S'(t)}} I_{ij}(t+1)$ and $\sum_{i \in S_H \cap S'(t), j \in S_V \cap \overline{S'(t)}} I_{ij}(t+1)$. We will show that the contact probability between any V-node and H-node is independent of the positions of their paths, thus, the latter two terms are independent of the segmentations before the move.

According to Fig. 4, for any $i \in S_H \cap S'(t)$ located x away from the vertical path of $j \in S_V \cap \overline{S'(t)}$, the probability that (i, j) is a contact pair is the proportion of the chord length $|AB|$ over the unit side length. Taking the integral over all i 's possible positions on the horizontal path, their contact probability p_{H-V} is

$$p_{H-V} = \int_{-r}^r 2\sqrt{r^2 - x^2} dx = \pi r^2,$$

which is independent of their positions.

Similarly, the contact probability between any $i \in S_V \cap S'(t)$ and $j \in S_H \cap \overline{S'(t)}$, p_{V-H} is

also πr^2 . Thus, the latter two terms can be estimated as

$$p(\mathbf{X}(t+1)|\mathbf{X}(t)) \left(\sum_{\substack{i \in S_H \cap S'(t), \\ j \in S_V \cap \overline{S'(t)}}} I_{ij}(t+1) \right) = p_{H-V} |S_H \cap S'(t)| |S_V \cap \overline{S'(t)}|,$$

and

$$p(\mathbf{X}(t+1)|\mathbf{X}(t)) \left(\sum_{\substack{i \in S_V \cap S'(t), \\ j \in S_H \cap \overline{S'(t)}}} I_{ij}(t+1) \right) = p_{V-H} |S_V \cap S'(t)| |S_H \cap \overline{S'(t)}|,$$

which are both independent of the segmentation between $S_H \cap S'(t)$ and $S_V \cap \overline{S'(t)}$.

Combined with the preceding discussion, the overall contact pair number is minimized through the combination of the two bottleneck segmentations within V-nodes and H-nodes, i.e. letting all V-nodes on the left half belong to $S'(t)$ and letting the right half of V-nodes belong to $\overline{S'(t)}$, and at the same time letting all H-nodes on the upper half belong to $S'(t)$ and the bottom half belong to $\overline{S'(t)}$, as shown in Fig. 5.

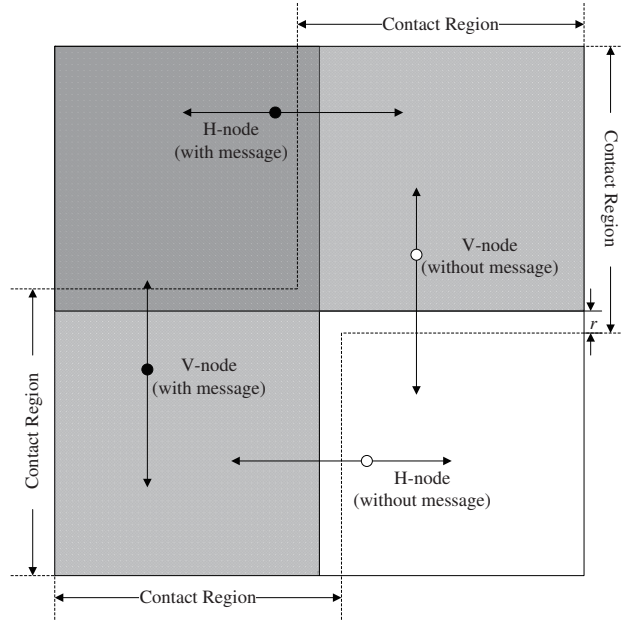


Fig. 5. One-dimensional Area Constrained Mobility

To sum up, the mobile conductance for one-dimensional mobility is:

$$\begin{aligned}
\Phi_m &= \left(\frac{n_V}{n}\right)^2 \Phi_s + \left(\frac{n_H}{n}\right)^2 \Phi_s + \min_{S'(t) \subset V(t), |S'(t)| \leq n/2} \left\{ \frac{P(r)}{|S'(t)|} \left(p_{H-V} |S'(t)| \frac{n_H}{n} \left| \overline{S'(t)} \right| \frac{n_V}{n} \right. \right. \\
&\quad \left. \left. + p_{V-H} |S'(t)| \frac{n_V}{n} \left| \overline{S'(t)} \right| \frac{n_H}{n} \right) \right\} \\
&= \frac{n_V^2 + n_H^2}{n^2} \Phi_s + \min_{S'(t) \subset V(t), |S'(t)| \leq n/2} \left\{ \frac{2n_V n_H |S'(t)|}{n^3} \right\} \\
&= \frac{n_V^2 + n_H^2}{n^2} \Phi_s + \frac{n_V n_H}{n^2}.
\end{aligned} \tag{18}$$

■

Remarks: We can see that, when all nodes move in one direction, the mobile conductance is the same as the static case. On the contrary, when half of the nodes are V-nodes and the other half are H-nodes, the mobile conductance achieves its maximum of $\Theta(1)$, the same order as in the fully random mobility model. The implication is that multidirectional movement spreads information faster than unidirectional movement.

2) *Two-dimensional Mobility:* The two-dimensional area-constrained mobility model is similar to the mobility model in [14], in which each node i has a *unique* home point i_h , and moves around the home point within a constrained disk of radius r_c . The home points are independently and uniformly distributed, forming a *random geometric graph* on the unit square. In each time slot, the position of a mobile node is uniformly distributed within the constrained circle area. Note that r_c is also called the *mobility capacity*.

Theorem 6: For the two-dimensional area-constrained mobility model defined above, where each node moves around its home point within a circle of radius r_c . When $r_c = w(r)$, the mobile conductance scales as $\Theta(r_c)$, and when the mobility capacity $r_c = o(r)$, the mobile conductance scales as $\Theta(r)$. If $r_c = \Theta(r)$, then mobile conductance scales both as $\Theta(r_c)$ and $\Theta(r)$.

Proof: Denote by $H_S \triangleq \{i_h\}, i \in S'(t)$ the set of home points for $S'(t)$, and $H_{\overline{S}} \triangleq \{i_h\}, i \in \overline{S'(t)}$ the set of home points for $\overline{S'(t)}$. Let X_{i_h} and X_{j_h} denote the positions of home points i_h and j_h , then i and j can possibly move to positions within a distance of r only if their home points are within a distance of $2r_c + r$, i.e., $E_{p(\mathbf{X}(t+1)|\mathbf{X}(t))} [I_{ij}(t+1)] > 0$ only if $|X_{i_h} - X_{j_h}| < 2r_c + r$. This is similar to the velocity constrained mobility model, except that the node's position before the move $X_i(t)$ is replaced by the position of its home point X_{i_h} , and v_{\max} is replaced by r_c .

Similarly, we should minimize $E[N_{S'}(t+1)]/|S'(t)|$ in order to achieve the minimum of mobile conductance. We now show that the two-dimensional area-constrained mobile conductance can be obtained similarly to the velocity constrained mobile conductance.

- 1) Here the node positions of $S'(t)$ and $\overline{S'(t)}$ after the move are not conditioned on their positions before the move, but determined by the positions of their home points. When calculating the expected number of contact pairs, H_S and $H_{\overline{S}}$ play the same roles as $S'(t)$ and $\overline{S'(t)}$ before the move, respectively.
- 2) The mobility capacity r_c has the same effect on information spreading as the maximal velocity v_{\max} in velocity constrained model, both of which set a limit on the nodes' moving ability.

Instead of finding the bottleneck segmentation between $S'(t)$ and $\overline{S'(t)}$ before the move as in the velocity constrained model, we need to find the bottleneck segmentation between the home points: H_S and $H_{\overline{S}}$. Since the home points also form a random geometric graph, we can use the same methods (See Appendix A) to find the bottleneck segmentation, which is dividing the home points into two halves (H_S and $H_{\overline{S}}$) using a straight vertical line bisecting the unit square, as illustrated in Fig. 6.

Therefore, we may follow the same line of evaluating the velocity constrained mobile conductance to obtain the two-dimensional area-constrained mobile conductance. We thus omit the details for the similarity. The only difference in the derived mobile conductance is that v_{\max} in (19) (in Appendix B) is replaced with the mobility capacity r_c , which leads to the final results in *Theorem 6*. ■

Remarks: The similarity between the two models is further validated by the simulation results (See Fig. 12 and Fig. 13). When the mobility capacity is much greater than transmission radius, the mobile conductance is dominated by the mobility capacity, i.e. $\Phi_m = \Theta(r_c)$. When the mobility capacity is much smaller than transmission radius, the mobile conductance is dominated by the transmission radius, i.e. $\Phi_m = \Theta(r)$, as in static networks.

V. SIMULATION RESULTS

In this section, we use large-scale simulations to verify the correctness and accuracy of the derived theoretical results.

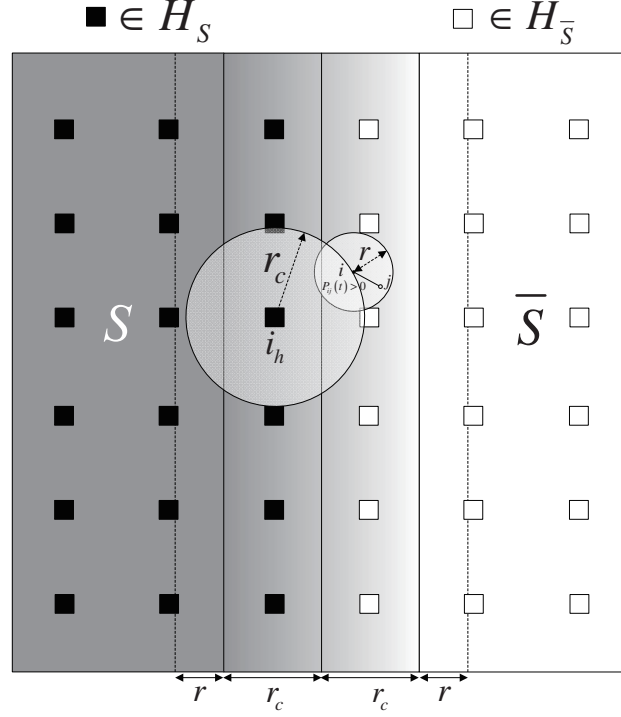


Fig. 6. Two-dimensional Area Constrained Mobility

The network and protocol models we simulate are described in Section II and the mobility models are presented in Section V. Specifically, all the n nodes are randomly deployed on a unit square and move according to certain mobility models. The transmission radius $r(n)$ is $\sqrt{\frac{C_0 \log n}{n}}$ and we set $C_0 = \frac{8}{\pi}$ [18]. The spreading time is measured by the number of time slots. For each curve, we simulate one thousand Monte-Carlo rounds and present the average.

The spreading time results for static networks and fully random mobile networks are shown in Fig. 7 to Fig. 10 as the upper and lower bounds. We observe that the spreading time in mobile networks is significantly reduced, and as network size n grows, the static spreading time increases much faster than the mobile counterpart. The bottommost curve (fully random mobility) grows in a trend of $\log n$ (note that the x-axis is on the log-scale), which confirms *Theorem 2*.

Fig. 7 further confirms our remarks on *Theorem 3*. When the proportion of mobile nodes is a constant ($0.1n$), the corresponding curve exhibits a slope almost identical to that for the fully random model. We also observe that $k = \Theta(\sqrt{n \log n})$ is a breaking point, below which ($k = \Theta(\sqrt{\log n})$) the performance degrades to the static case.

Fig. 8 confirms our remarks on *Theorem 4*. When $v_{\max} = 0.1$, the corresponding curve exhibits

a slope almost identical to that for the fully random model. We also observe that $v_{\max} = \Theta(r)$ is a breaking point, any velocity that is lower ($v_{\max} = n^{-\frac{1}{2}}$) leads to a performance similar to the static case.

The spreading time results for the one-dimensional area constrained mobility model is shown in Fig. 9, which exhibit slopes almost identical to that for the fully random model. It is also shown that when half of the nodes are V-nodes and the other half are H-nodes, the best performance is achieved.

Fig. 10 confirms our remarks on *Theorem 6*. When $r_c = 0.1$, the corresponding curve exhibits a slope almost identical to that for the fully random model. We also observe that $r_c = \Theta(r)$ is a breaking point, any mobility capacity that is lower ($r_c = n^{-\frac{1}{2}}$) leads to a performance similar to the static case.

Fig. 11 compares the conductances of static and mobile networks. The fully random mobile conductance scales as $\Theta(1)$. However, the static conductance drops with the network size n , and roughly fits in a $\sqrt{\frac{\log n}{n}}$ scaling. Between the two curves is the partially random mobile conductance ($k = 0.01n$), which is the combination of the two extreme cases. Again, all the curves validate our theoretical results.

In the following simulations, we fix the network size as 1000, and measure the mobile conductances under various parameters. Fig. 12 shows the velocity constrained mobile conductance. When $v_{\max} \ll r$, the mobile conductance keeps steady, which corresponds to the static conductance. When $v_{\max} \gg r$, the mobile conductance grows approximately linearly with v_{\max} until approaching the fully random mobile conductance illustrated in Fig. 11.

Fig. 13 shows the mobile conductance for the two-dimensional area constrained mobility model. Note that Fig. 13 is nearly identical with Fig. 12, which corroborates our analysis.

VI. CONCLUSION

In this paper, we analyze information spreading in mobile networks, based on the proposed ‘move-and-gossip’ information spreading model. We first derive the spreading time in mobile networks with respect to the newly defined metric mobile conductance, and show that mobility can speed up information spreading. Then, we describe four types of mobility models (fully random mobility, partially random mobility, velocity constrained mobility and area constrained mobility), and analyze the effects of various mobility patterns on mobile conductance and

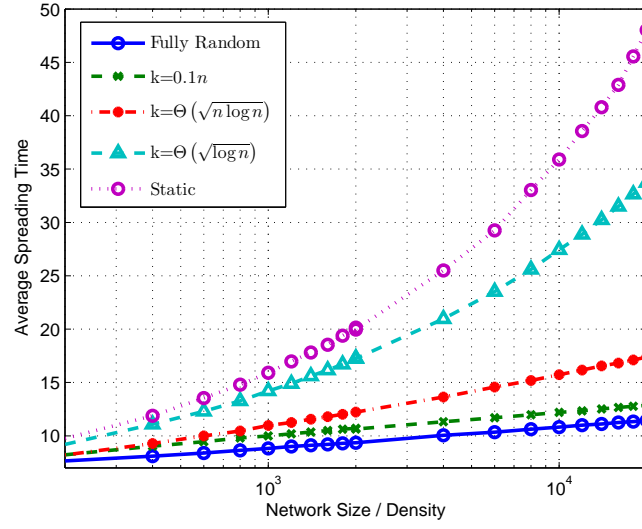


Fig. 7. Average Spreading Time under the Partially Random Mobility Model

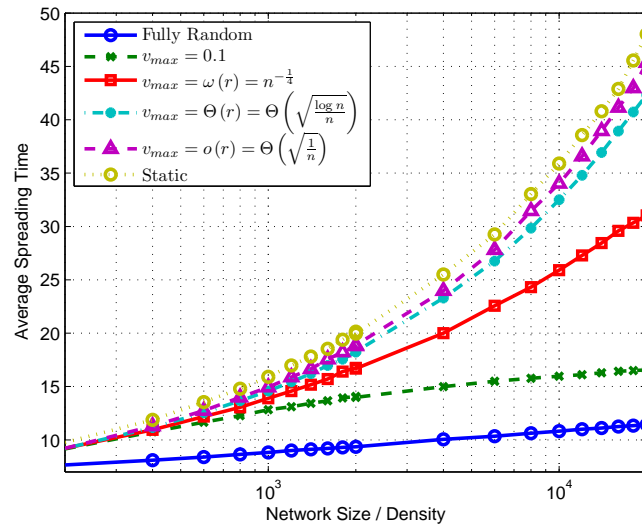


Fig. 8. Average Spreading Time under the Velocity-Constrained Mobility Model

information spreading. Finally, we provide comprehensive simulation results to supports our theoretical analysis.

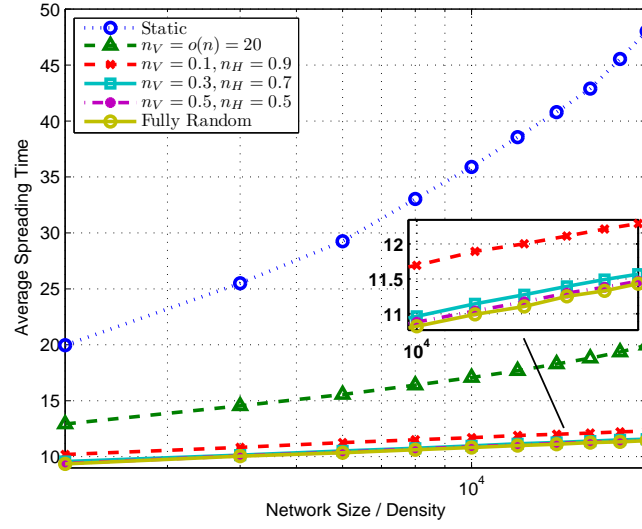


Fig. 9. Average Spreading Time under the One-Dimensional Area Constrained Mobility Model

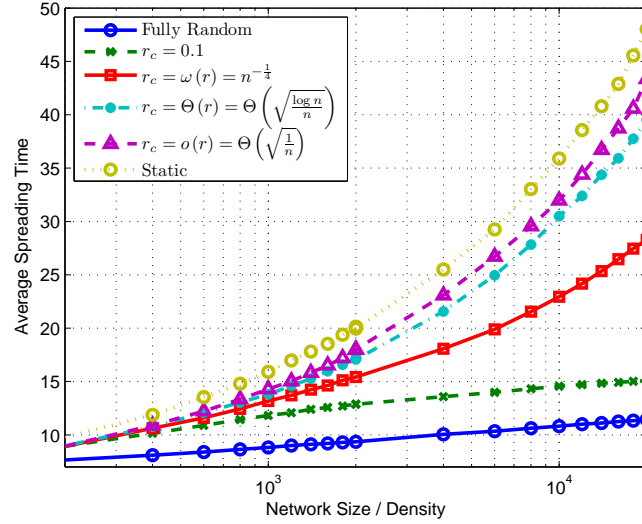


Fig. 10. Average Spreading Time under the Two-Dimensional Area Constrained Mobility Model

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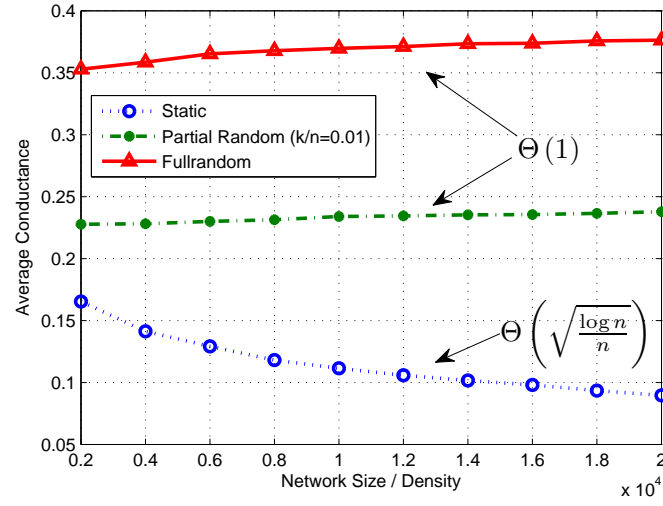


Fig. 11. Comparisons of Average Conductance between Static and Mobile Networks

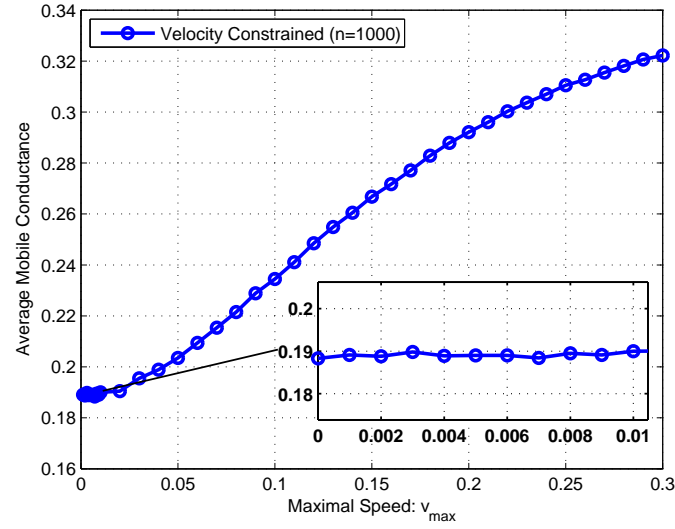


Fig. 12. Average Conductance of Velocity Constrained Mobility

Funds for the Central Universities of China, and the US National Science Foundation under Grants CCF-0830462 and ECCS-1002258.

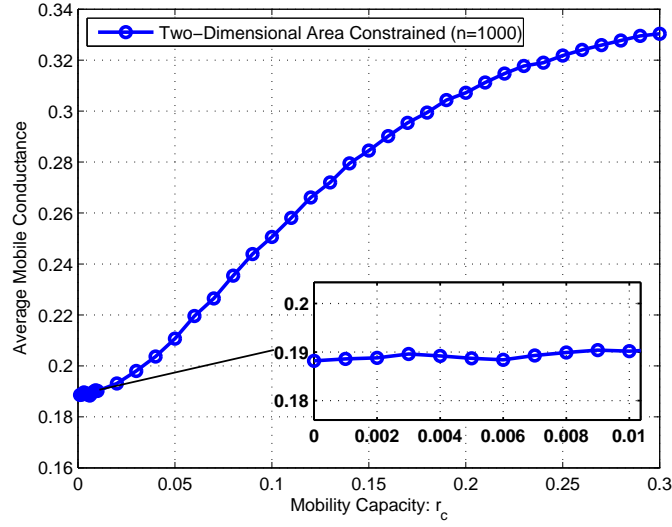


Fig. 13. Average Conductance of Two-Dimensional Area Constrained Mobility

APPENDIX A

BOTTLENECK SEGMENTATION FOR VELOCITY CONSTRAINED MOBILITY MODEL

Lemma 1: The minimum in mobile conductance under velocity constrained mobility is achieved through the *bottleneck segmentation* using a vertical straight line bisecting the unit square.

Proof: Nodes at most $2v_{max} + r$ away from the border are able to move and contact the

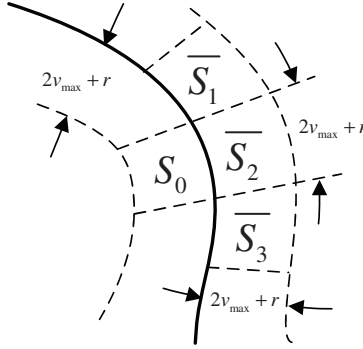


Fig. 14. The border in velocity constrained mobile networks

nodes on the other side of the border. Denote by $B(t)$ the length of border and partition the area near the border into bins of area (approximately) $S_b = (2v_{max} + r)^2$ as in Fig. 14. The nodes in a bin can contact nodes in three bins on the opposite side. For example, the nodes in S_0 can

reach $\overline{S_1}$, $\overline{S_2}$, and $\overline{S_3}$ after the move. Given v_{\max} , r and n , the number of possible contact pairs for S_0 is on the order of $\Theta(n^2 S_b^2)$. The total number of contact pairs after the move is

$$N_{S'}(t+1) = \frac{B(t)}{2v_{\max} + r} \Theta(n^2 S_b^2).$$

Therefore, the number of contact pairs after the move is proportional to the length of the border before the move, i.e. $N_{S'}(t+1) \propto B(t)$, and the mobile conductance is $\Phi_m \propto \frac{B(t)}{|S'(t)|}$. Following the same argument in static conductance [5], the ratio of $\frac{B(t)}{|S'(t)|}$ is minimized by a vertical straight line bisecting the unit square. Therefore, the mobile conductance under velocity constrained mobility is also minimized through this *bottleneck segmentation*. ■

APPENDIX B

EVALUATION OF VELOCITY CONSTRAINED MOBILE CONDUCTANCE

The accurate evaluation in (16) over the circle is rather involved, therefore we loosen the requirement by only calculating over the small dashed square in the circle, as illustrated in Fig. 3. Specifically, we replace (16) with

$$E_{p(\mathbf{X}(t+1)|\mathbf{X}(t))} [N_{S'}(t+1)] \cong \int_{-v_{\max}-r}^{v_{\max}+r} \rho'_{S'}(x) \int_{x-\frac{r}{\sqrt{2}}}^{x+\frac{r}{\sqrt{2}}} \rho'_{\bar{S}'}(l) \sqrt{2}r dl dx.$$

This will result in a smaller mobile conductance, but the scaling law will not be affected in the order sense. After the integral, the mobile conductance is approximated by

$$\Phi_m(\mathcal{P}_{\mathbf{X}}) \cong \begin{cases} \frac{1}{2}r + \frac{v_{\max}^2}{3r}, & \text{for } v_{\max} \leq \frac{1}{2}r, \\ -\frac{r^3}{48v_{\max}^2} + \frac{r^2}{6v_{\max}} + \frac{2}{3}v_{\max}, & \text{for } v_{\max} > \frac{1}{2}r. \end{cases} \quad (19)$$

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